How to quickly cool a glass?
The Mpemba effect in rugged landscapes

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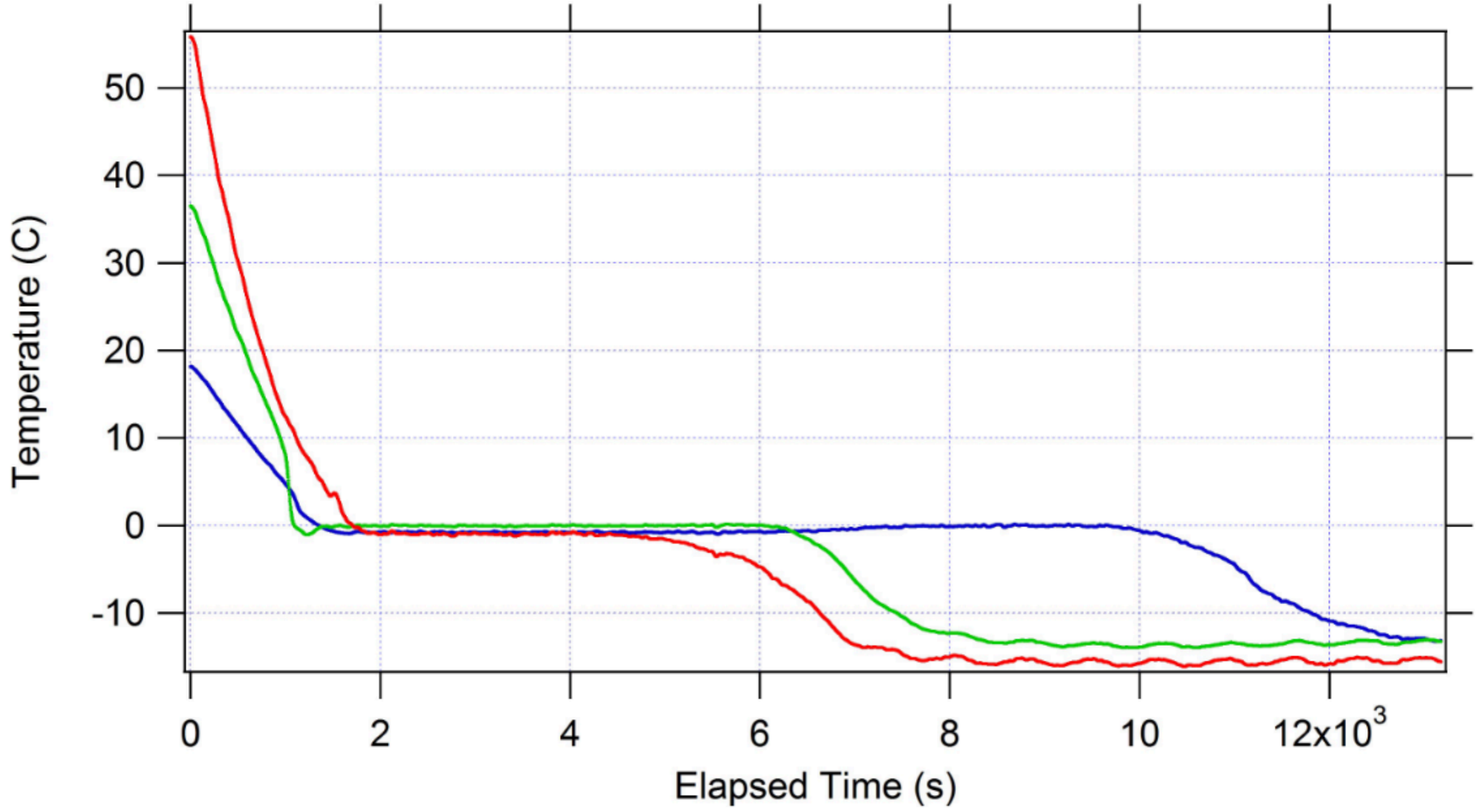
References

- Z. Lu and O. Raz
  Nonequilibrium thermodynamics of the Markovian Mpemba effect and its inverse
  Proceedings of the National Academy of Sciences 114.20 (2017): 5083-5088

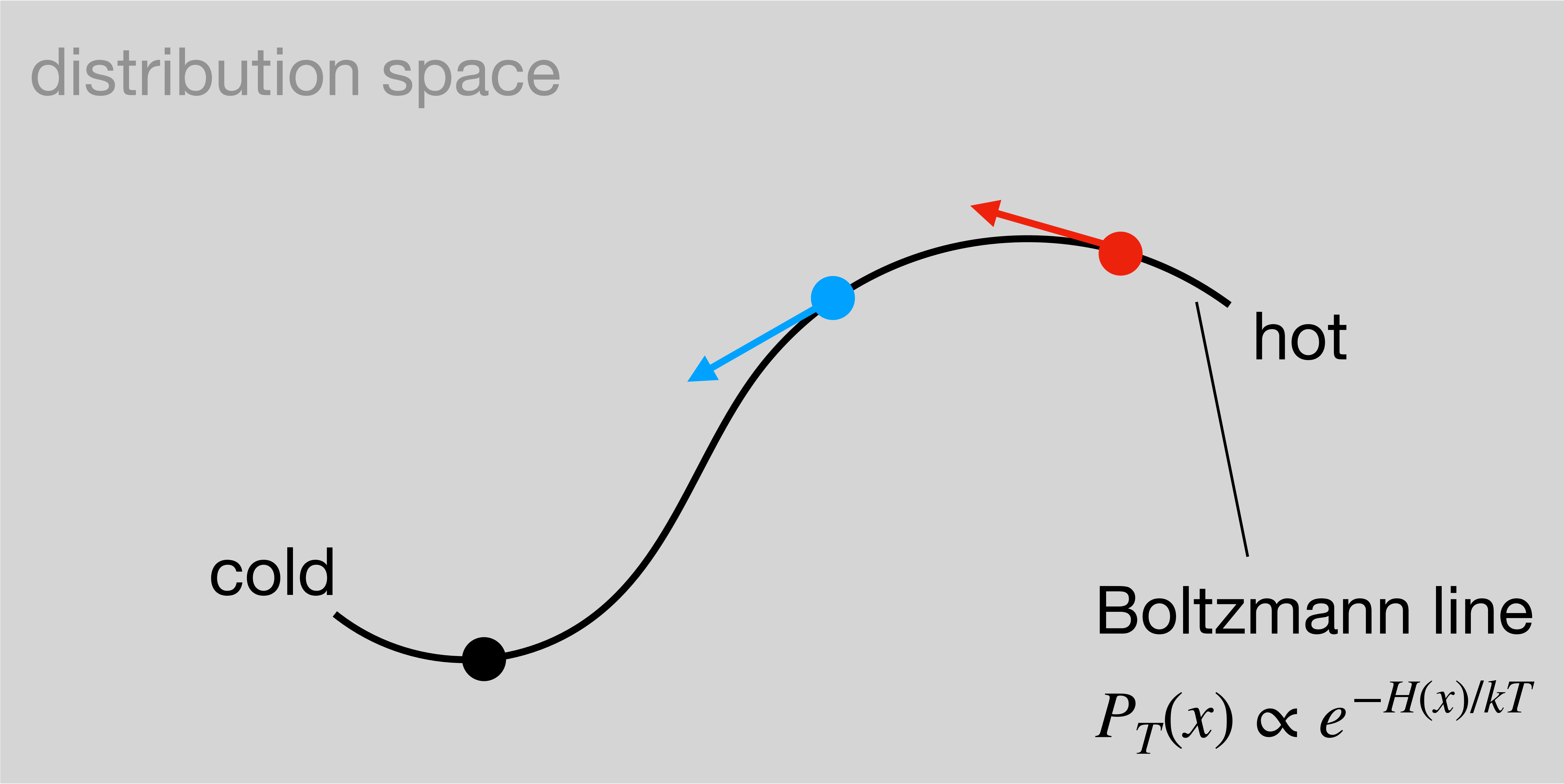
- M. Baity-Jesi et al.
  The Mpemba effect in spin glasses is a persistent memory effect
  Proceedings of the National Academy of Sciences 116.31 (2019): 15350-15355

The Mpemba effect in water

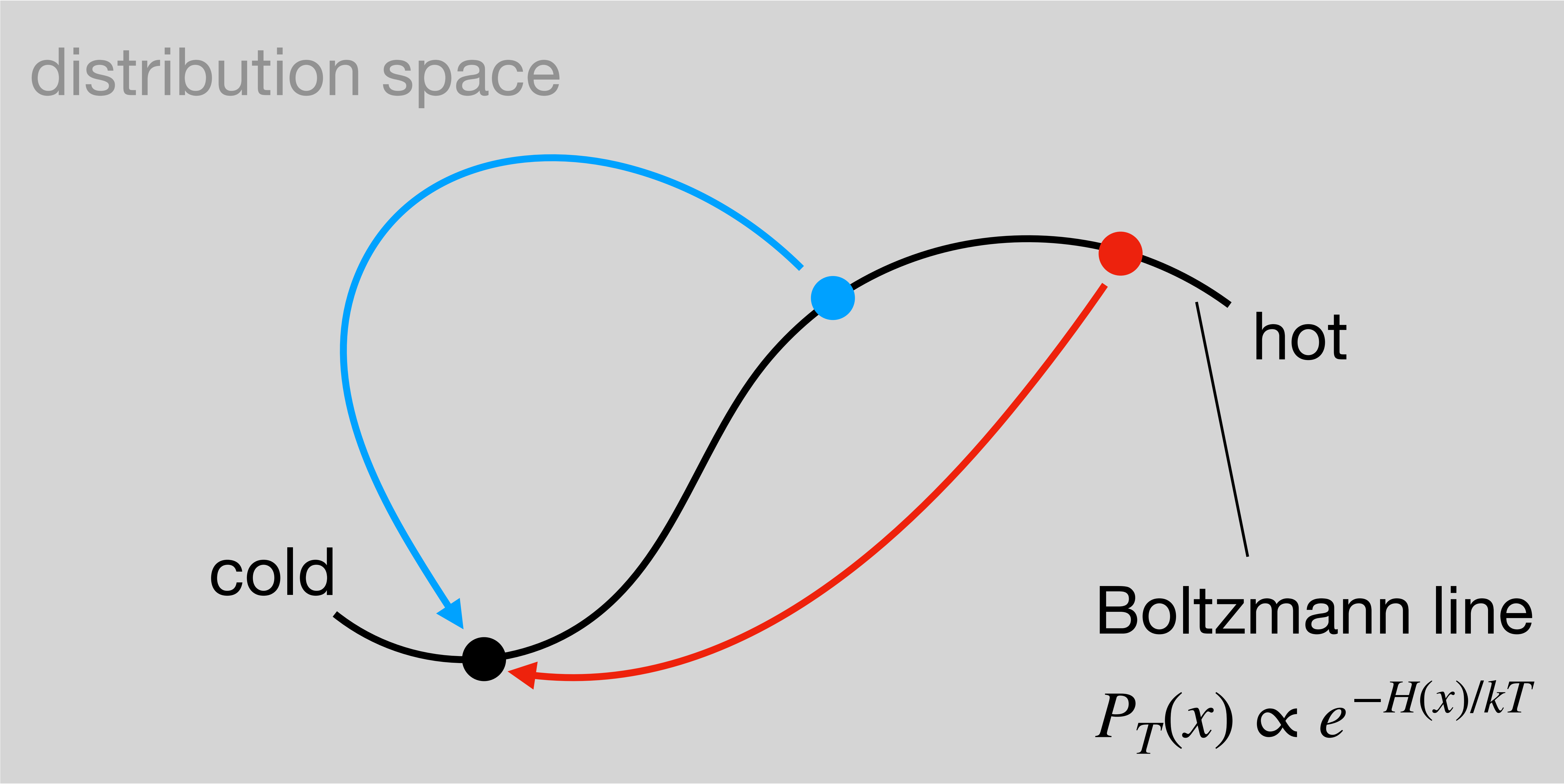


[J. H. Thomas, Wooster Open Works (2007)]

The quasi-static origin of the paradox



Non-equilibrium shortcuts



Non-equilibrium Markovian dynamics

- Markov property implies master equation

$$\frac{dp_i(t)}{dt} = \sum_j R_{ij}(T_b) p_j(t)$$

- Assuming ergodicity + detailed balance:
  $R_{ij}(T_b)$ is diagonalizable with real eigenvalues

$$\lambda_0 = 0 > \lambda_1 > \ldots > \lambda_n$$

  and (right) eigenvectors

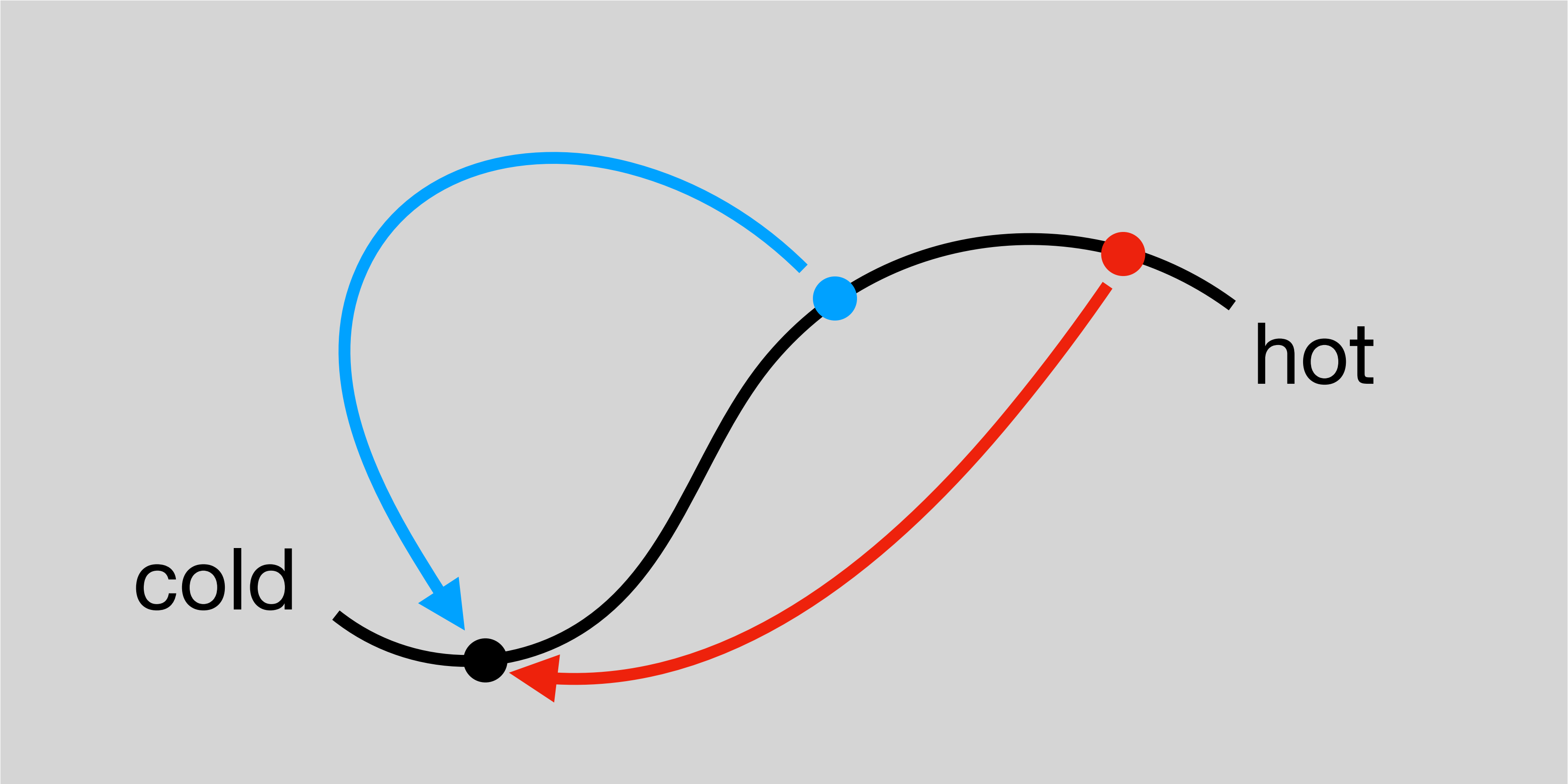
$$v_0 = p_{eq}\,,\ v_1,\ \ldots\ ,v_n$$

The Markovian Mpemba effect

- Starting conditions: $p^h(0)$ and $p^c(0)$
- After quench, relaxation to equilibrium is identical at long times

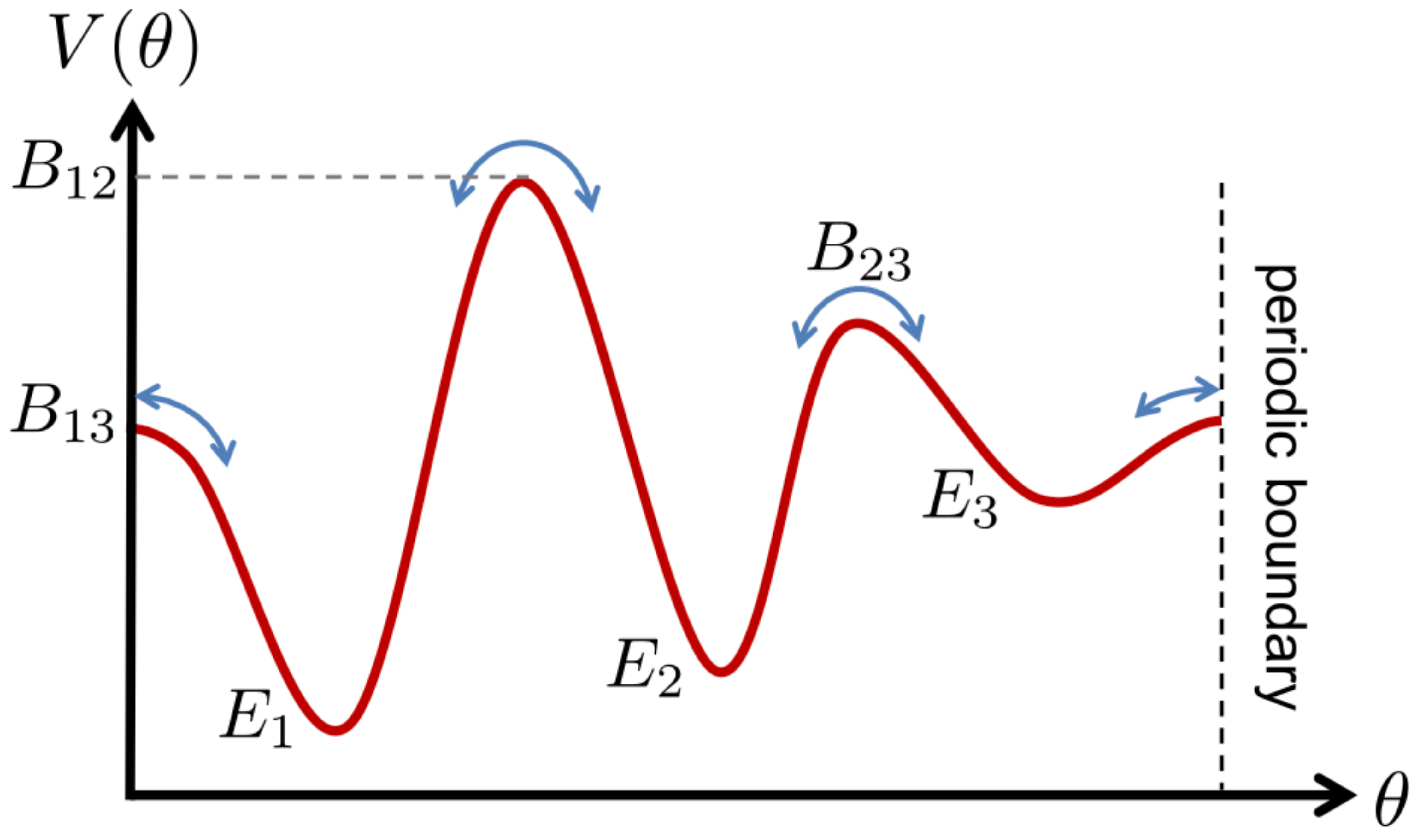
$$p^h(t) = p_{eq} + a_1^h e^{\lambda_1 t} v_1 + \dots$$

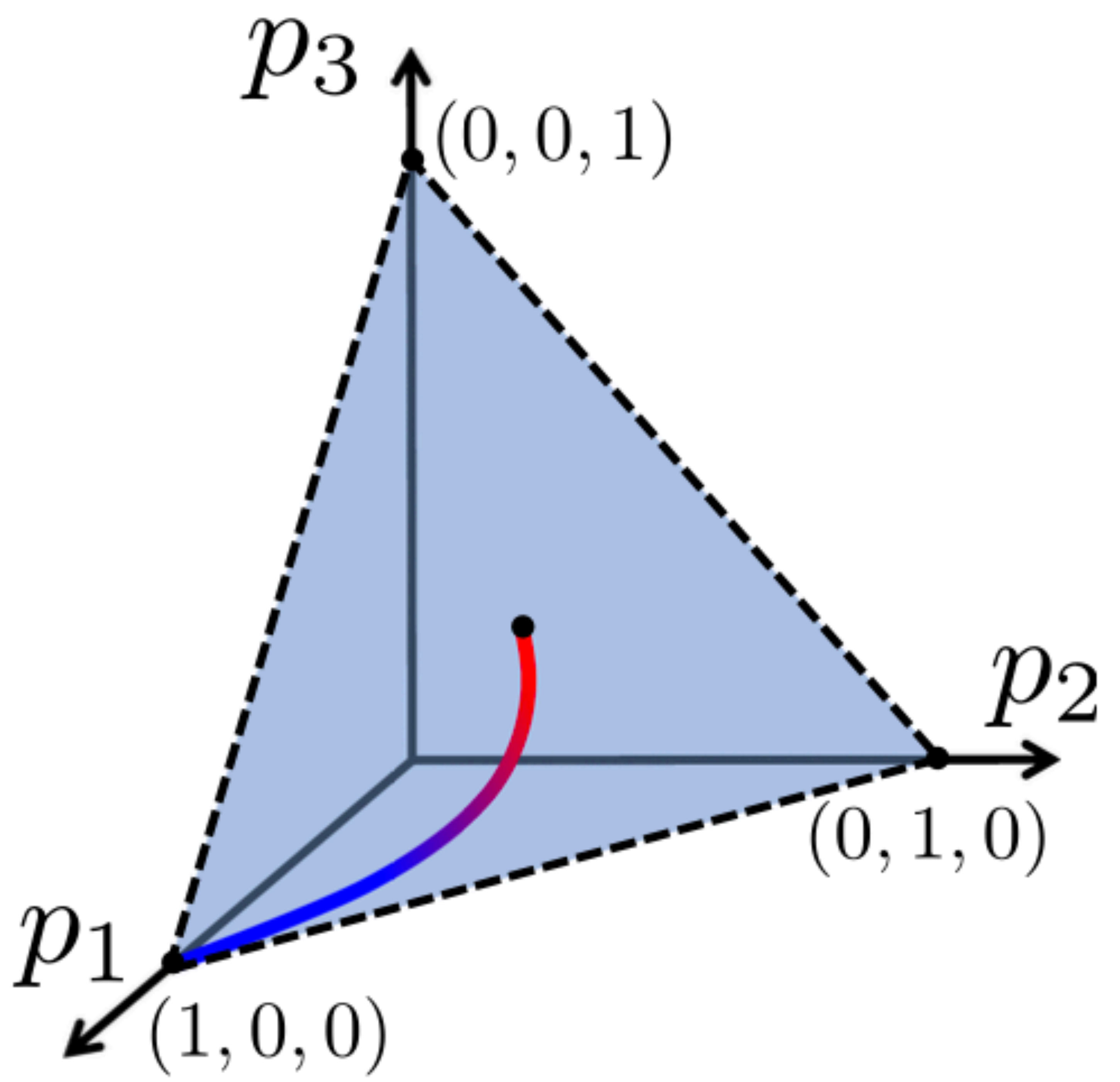
$$p^c(t) = p_{eq} + a_1^c e^{\lambda_1 t} v_1 + \dots$$



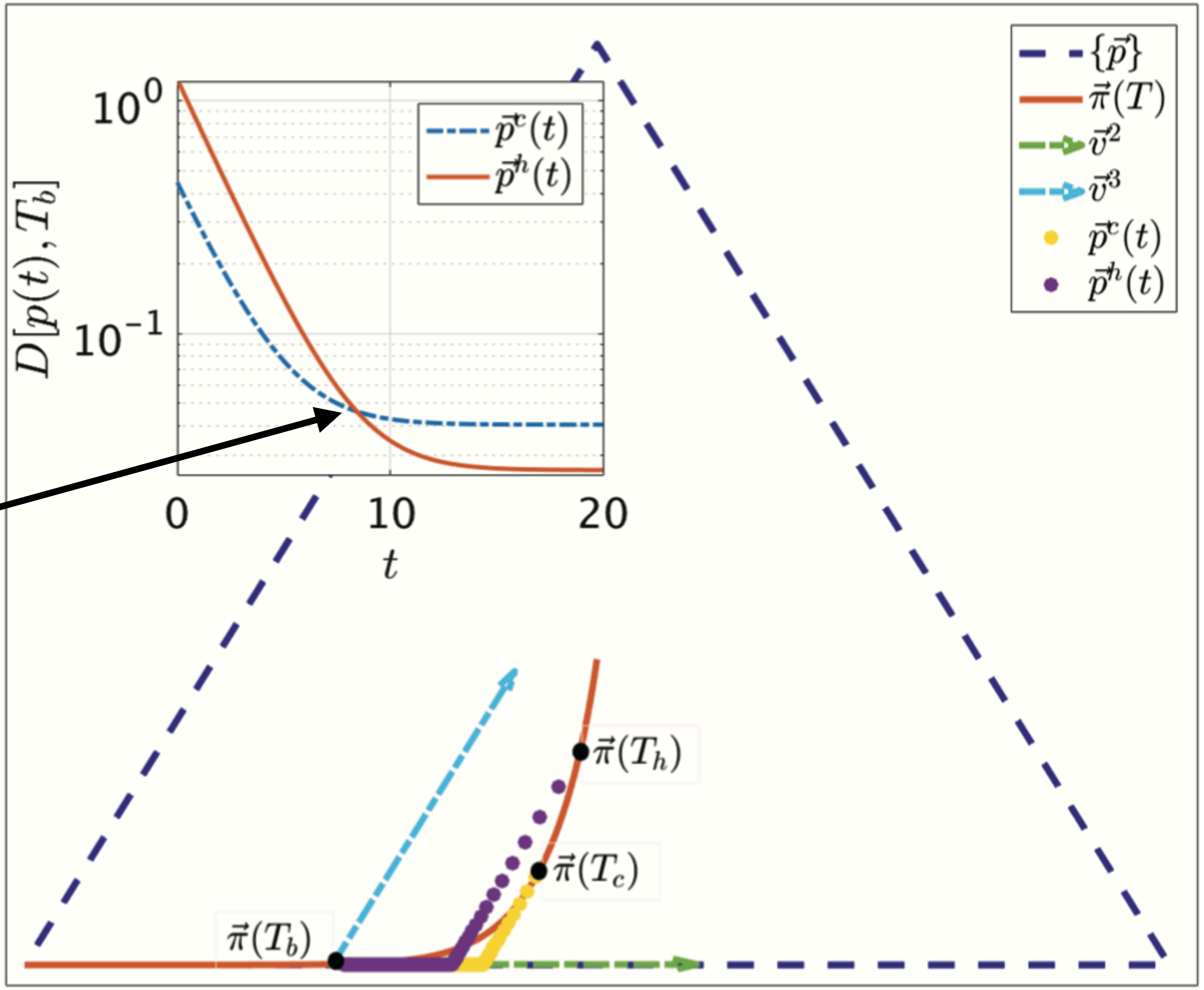
- If $a_1^c > a_1^h$ then the cold system lags behind the hot one: Mpemba effect!
- If $a_1 = 0$, relaxation is “exponentially” faster: “strong” Mpemba effect

Example: three-state system



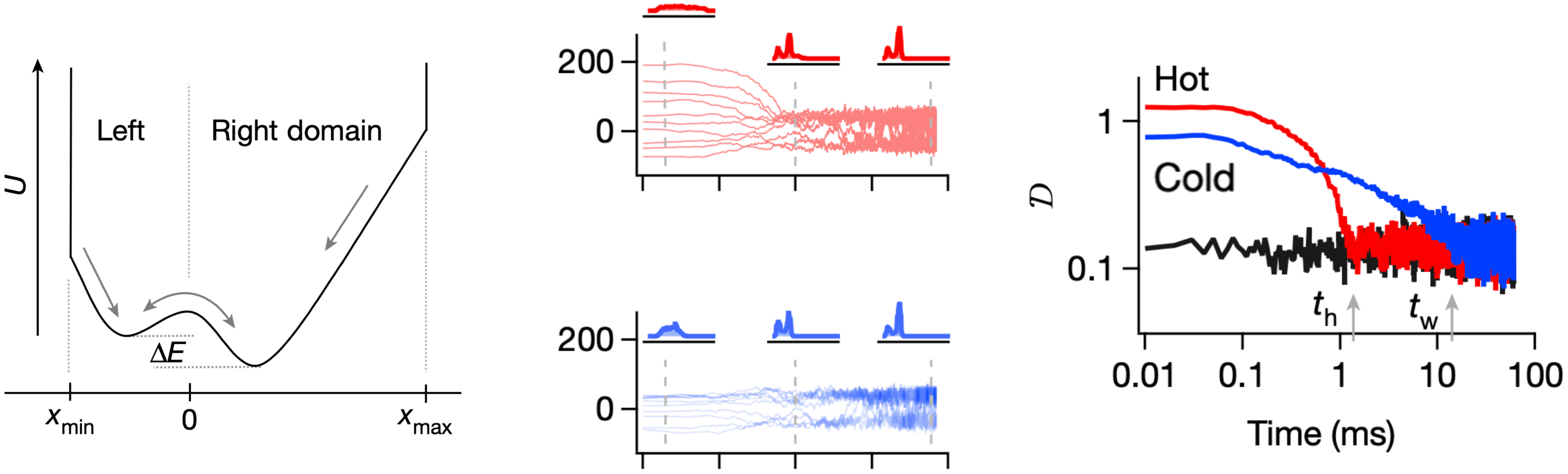


Example: three-state system



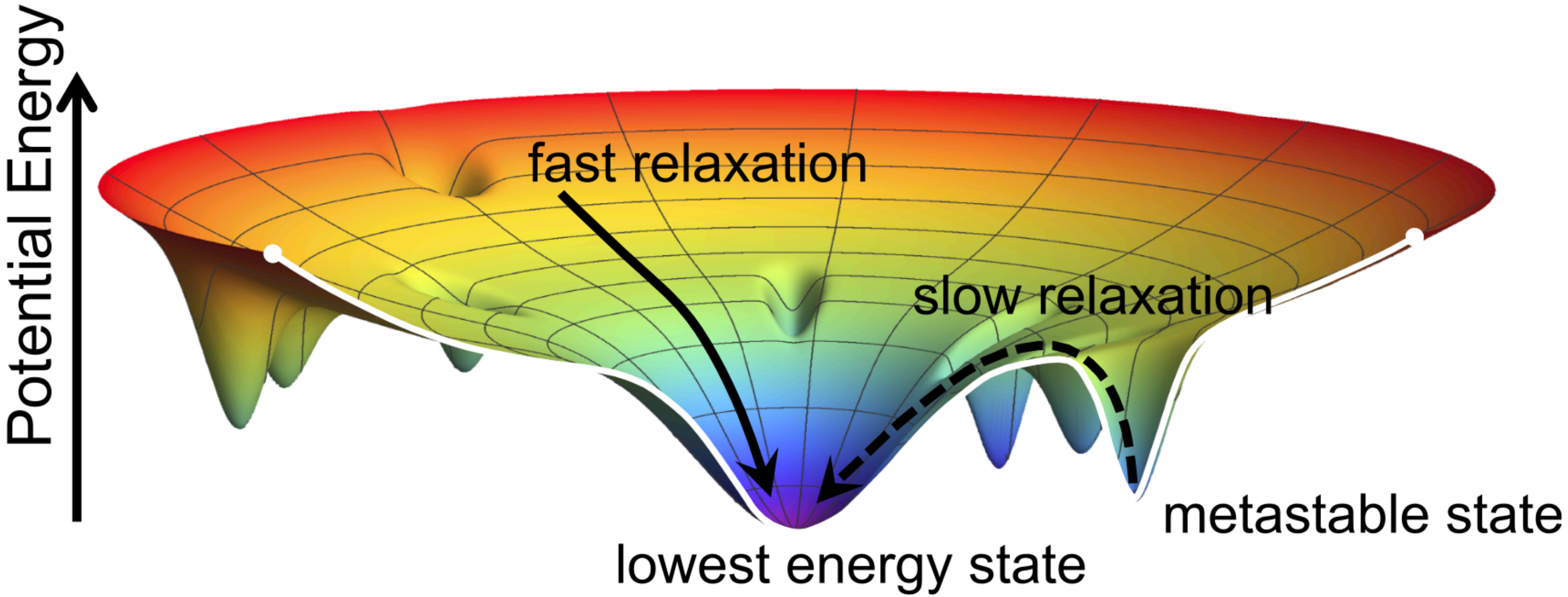
Mpemba effect

The first experiment: a colloid in a double-well



[Kumar and Bechhoefer, Nature 2020]

Mpemba effect and rugged landscapes

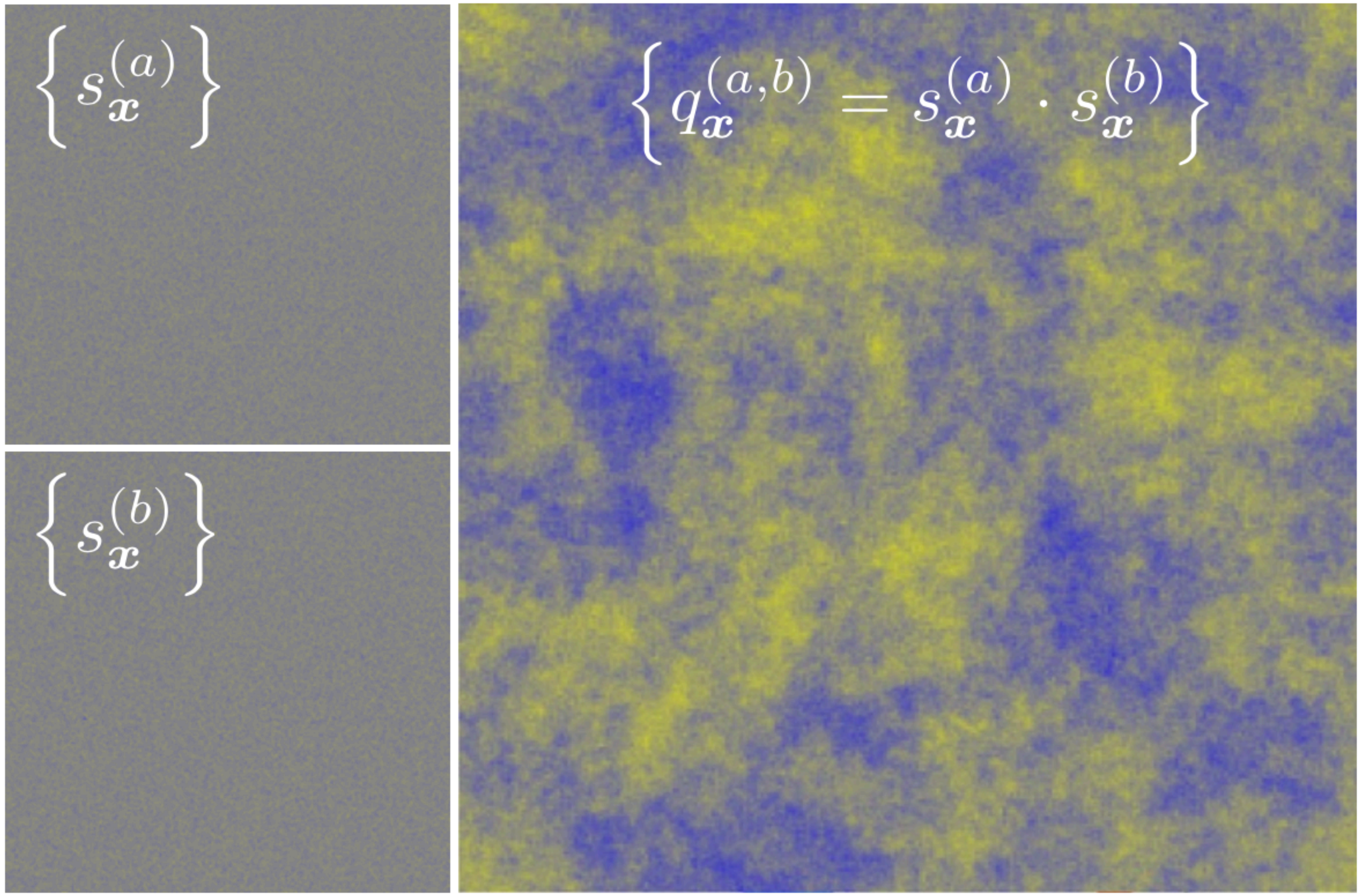


A simple spin-glass

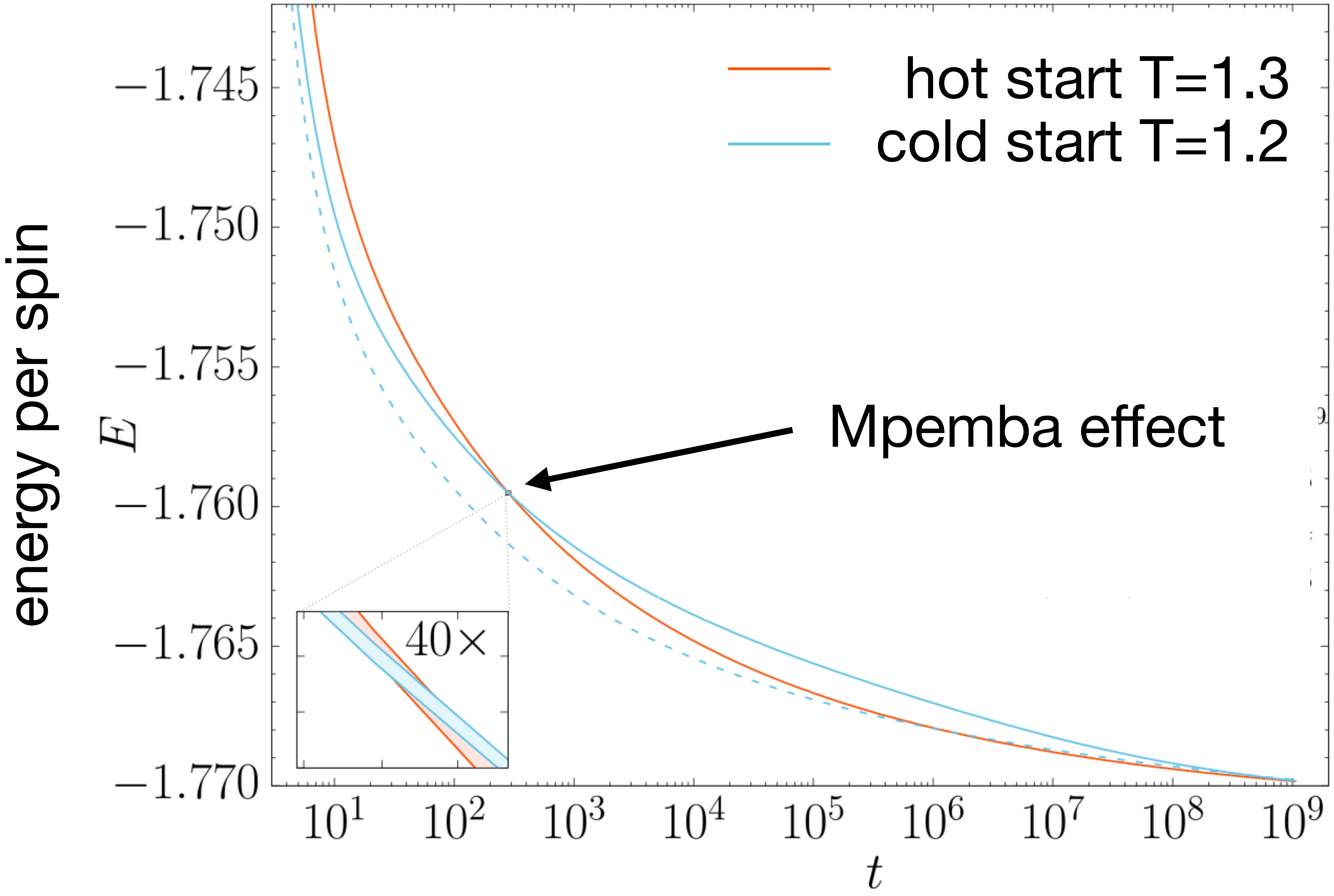
$$H = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j \qquad J_{ij} = \pm 1$$

- In 3D, spin-glass transition at $T_c = 1.102(3)$
- Simulation details
  - 3D with $L = 160$
  - 16 samples of $\{J\}$, 256 replicas each
  - Metropolis algorithm, 1 lattice sweep $\approx$ 1 ps
  - simulation time from 1 ps to 0.1 s (!!)

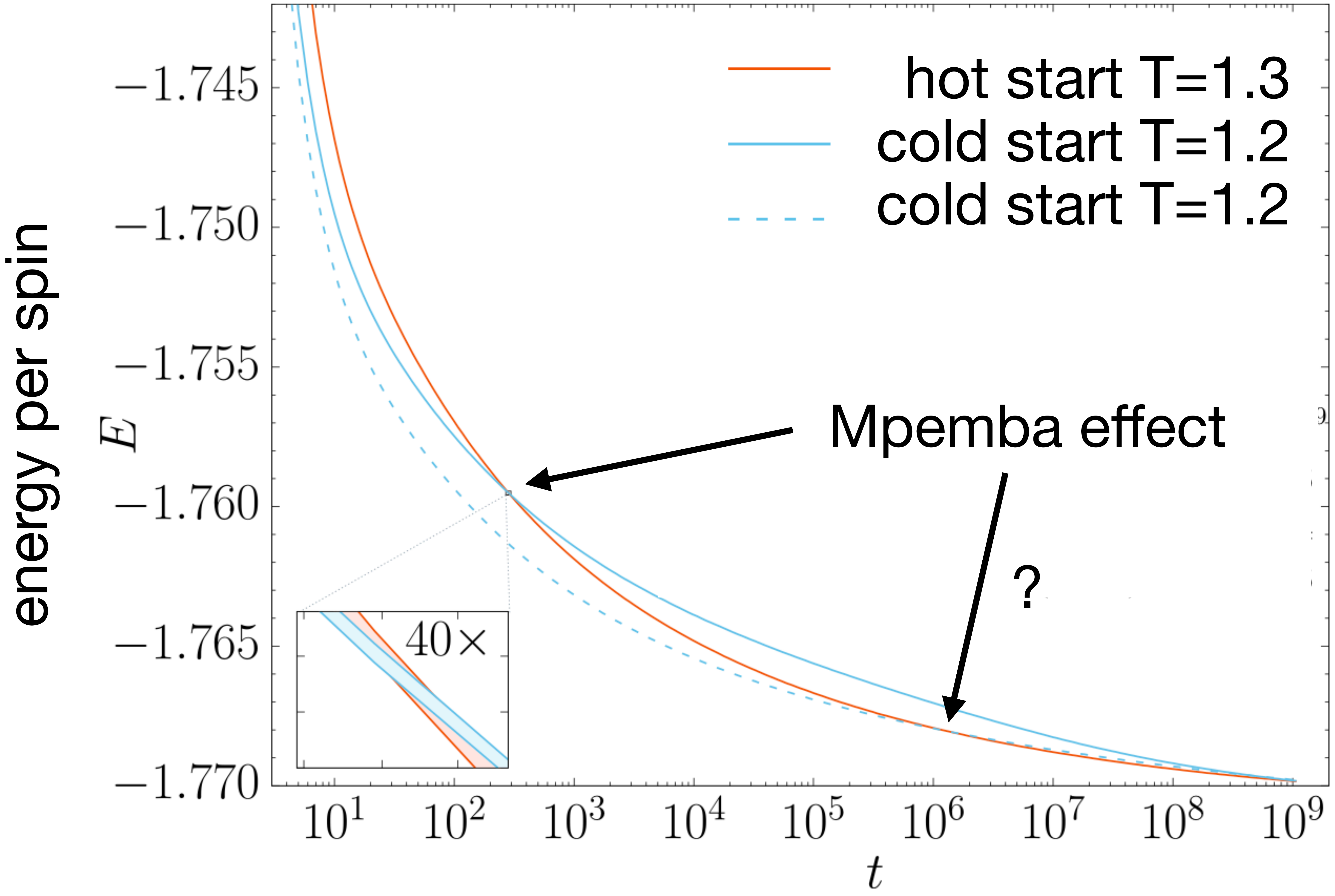
A realization at $T < T_c$



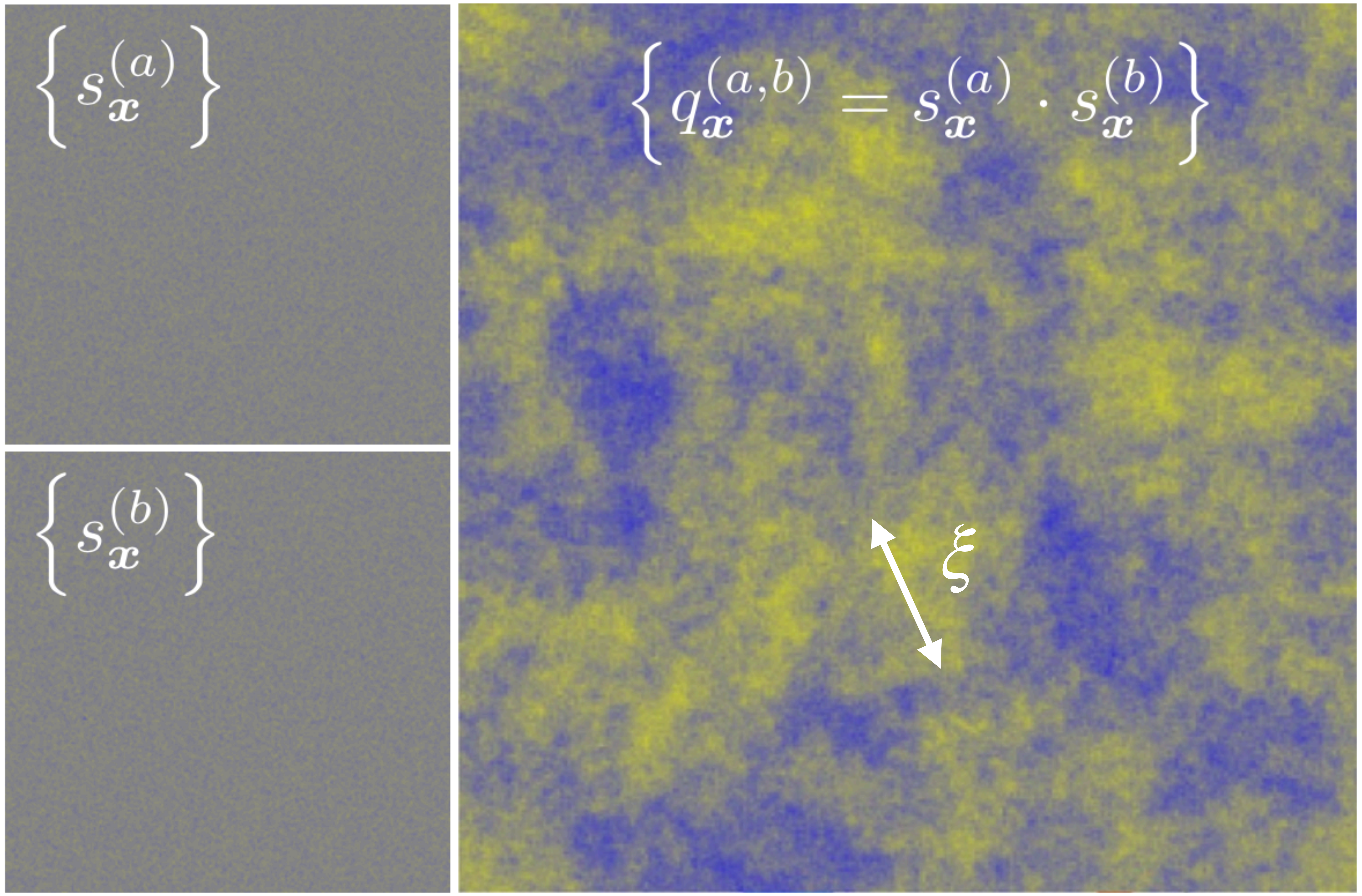
Relaxation after a quench below T_c



Relaxation after a quench below T_c



Coherence length



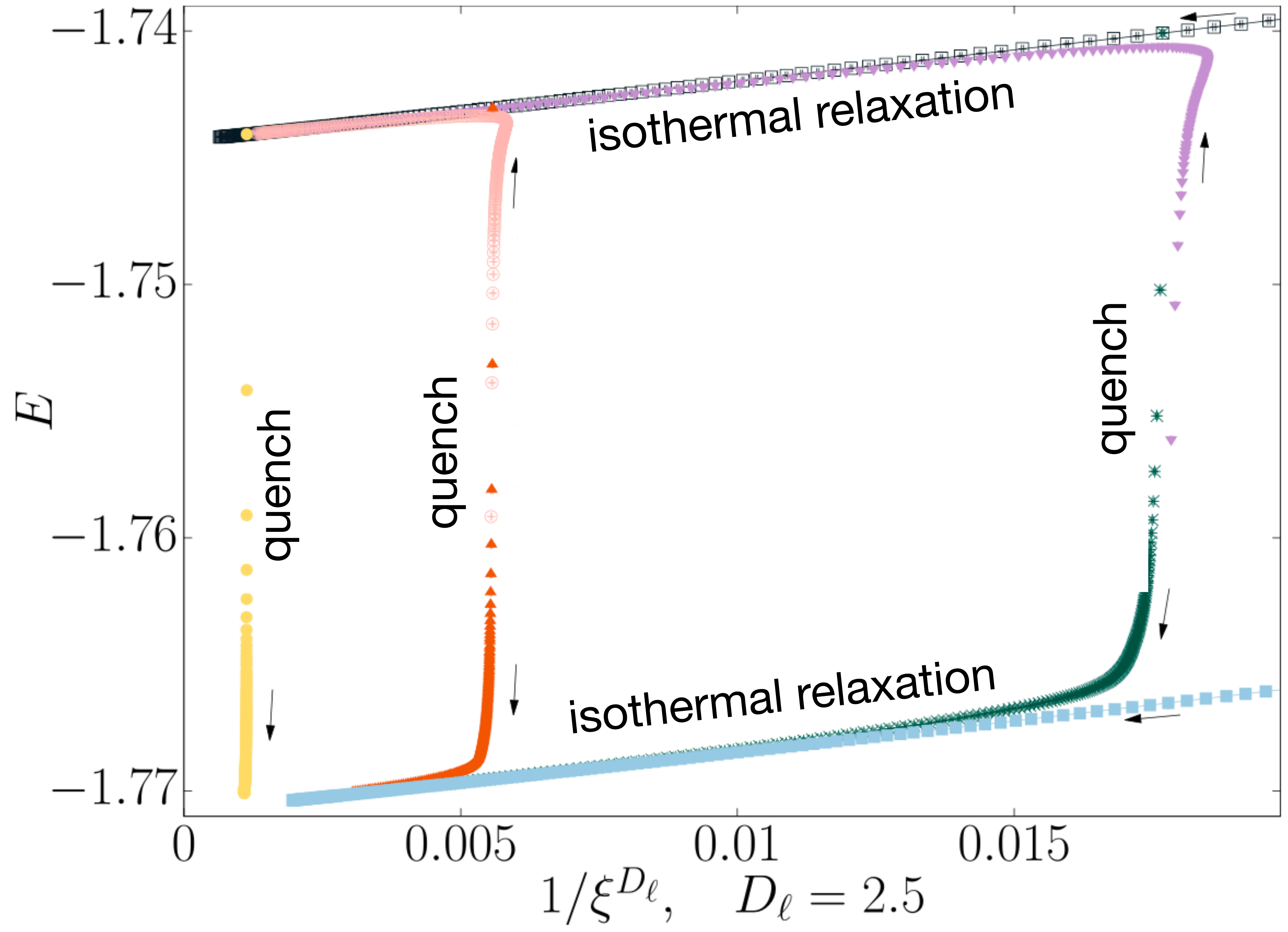
- Above T_c grows (slowly) to equilibrium value
- Below T_c grows without bounds (slowly) as $\xi \sim t^{1/z(T)}$

How to quickly cool a glass?

$$E(t) = E_\infty(T) + \frac{E_1}{\xi(t)^{D_l}} + \ldots$$

$$D_l \approx 2.5$$

lower critical dimension



What about ecological communities?

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Editors' Suggestion

Properties of Equilibria and Glassy Phases of the Random Lotka-Volterra Model with Demographic Noise

$$\frac{dN_i}{dt} = N_i\left[1 - N_i - \sum_{j,(j\neq i)} \alpha_{ij}N_j\right] + \eta_i(t)$$

$$\langle \eta_i(t)\eta_j(t')\rangle = 2TN_i(t)\delta_{ij}\delta(t-t')$$